**Forecasting Retailer Product Sales in The Presence of Structural Change**

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Abstract

Grocery retailers need accurate forecasts at the Stock Keeping Unit (SKU) level for their inventory management decisions. Previous studies have developed forecasting models which incorporate the effect of various marketing activities including prices and promotions. These models, however, assume that the effect of these marketing activities on product sales to be constant over time. They may potentially be subject to the structural change problem as they are unable to capture the varying effect of the marketing activities. As a result, they could generate biased and less accurate forecasts. In this study, we propose new forecasting methods for retailer product sales which take into account the problem of structural change. Our methods generate more accurate forecasts compared to conventional models which assume constant parameters for various marketing activities.

Keywords:

Forecasting, OR in marketing, Analytics, Retailing

1. **Introduction**

Grocery retailers rely on accurate sales forecasts for their inventory management (Petropoulos, Makridakis, Assimakopoulos, & Nikolopoulos, 2014). Poor forecasts of product sales lead to poor service arising from out-of-stock conditions or, alternatively, inflated costs due to overstocking. When a specific item is out-of-stock, retailers directly lose the income and profit from the sale of the item. If the out of stock situation happens on a regular basis, it can lead to consumer dissatisfaction. In the long term, retailers may see customers switching to other retail chains (Corsten & Gruen, 2003). To avoid such situations, retailers may intentionally overstock to maintain a high customer satisfaction level but this significantly raises inventory costs (e.g., capital cost, warehousing, and deterioration etc.) and reduces profits (L. Cooper, Baron, Levy, Swisher, & Gogos, 1999). In 2014, retailers in North America had a loss of $634.1 billion due to out-of-stock and spent $471.9 billion on overstock (OrderDynamics, 2015). One of the solutions to mitigate the dilemma is to generate more accurate sales forecasts at Stock Keeping Unit (SKU) level, which improves the effectiveness of the supply chain management by reducing the bullwhip effect and enabling the Just-In-Time delivery (Ouyang, 2007; Sodhi & Tang, 2011).

In practice, many retailers generate forecasts at SKU level using a two-stage ‘base-lift’ approach. The forecasts are generated separately depending on whether or not the focal product is being promoted. The ‘base’ forecasts are generated using simple univariate models, while the ‘lift’ effect, which is effectively caused by marketing activities including price reductions and promotions, is estimated by the brand/category manager based on his/her experience. In this context, some previous studies have proposed procedures to help managers improve the accuracy of their judgments (e.g., Fildes, Nikolopoulos, Crone, & Syntetos, 2008; Goodwin, 2002; Nikolopoulos, 2010). Others have developed models to estimate the ‘lift’ effect based on data (L. Cooper et al., 1999; L. G. Cooper & Giuffrida, 2000; Trusov, Bodapati, & Cooper, 2006). A third type of approach develops methods to directly generate the final forecasts of the product sales. For example, Gür Ali, SayIn, van Woensel, and Fransoo (2009) proposed the regression tree method with a range of variables constructed from the sales, price, and promotion of the focal product. Huang, Fildes, and Soopramanien (2014) proposed two-stage general-to-specific Autoregressive Distributed Lag (ADL) models which incorporated the promotional information of not only the focal product but also of the competitive products within the same product category. Ma, Fildes, and Huang (2016) further integrated the promotional information of the products from related product categories.

All these studies assume that the impact of marketing activities on product sales remains constant over time. In practice, the effect of prices and promotions may change due to the many non-controllable factors which may include, for instance, changing economic conditions, changes in consumer tastes, and the entry of new competitors etc., some of which are neither observable or measurable (Wildt, 1976; Wildt & Winer, 1983). Customers may become more sensitive to prices and promotions during an economic crunch. They may change their tastes due to factors including their familiarity with the product, and their changing lifestyle and social status (Meeran, Jahanbin, Goodwin, & Quariguasi Frota Neto, 2017). When a new competitor enters the market, the effect of prices and promotions of the focal product may be reduced not only because the new competitor launches their marketing activities but also because customers seek variety. In the year of 2014, the German low-price retail chain Aldi opened more than 400 stores in the United States, leading to changes in customer grocery purchasing habits, which then put pressures on existing retail chains (Loeb, 2014).

Under any of the circumstances described above, conventional models which use constant parameters to represent the effect of the price and promotions may potentially be subject to the structural change problem (Allen & Fildes, 2001; Armstrong, 2001). As a result, the forecasts generated by these models may be biased and less accurate. The structural change problem has been well addressed by previous studies (see a summary in M. P. Clements & Hendry, 1999). The problem of the structural change has been overlooked in forecasting retailer product sales. In this study, we propose new effective methods to generate more accurate forecasts by taking into account the problem of structural change. Specifically, we propose the Autoregressive Distributed Lag (ADL) models with Intercept Correction which estimate the bias back to the forecasts, and the ADL model and with Estimation Window Combining which resorts to forecast combination to achieve an effective trade-off between the forecast bias and the forecasting error variance.

Our research in the domain of retail forecasting in particular at SKU level is significant for the following reasons. First, our research is the first research which investigates the structural change problem in forecasting retailer product sales. The data in retailer product sales at SKU level exhibit unique characteristics compared to data in other areas (e.g., macroeconomics). Also, the methods which deal with the structural change problem by reducing the associated forecast bias come with the cost of inflated forecast error variance (which also affects the forecasting accuracy). Under this circumstance, whether or not we can improve the forecasting accuracy by dealing with the structural change problem becomes an empirical question. The final results indicate that our models have superior forecasting performance compared to conventional models which assume no change in the effect of product prices and promotions. Second, unlike earlier studies which rely on incorporating additional information on the marketing activities (which leads to additional cost), our methods rely on how limited promotional information could be effectively utilized. In practice, the change of the effect of the marketing activities may be caused by many factors (as mentioned above) for which the data are difficult or infeasible to collect or measure. Therefore, our methods add value without incurring additional costs to retailers. Third, our research provides an evaluation of various forecasting methods which offers operational guidance to not only retailers but also to manufacturers when competitive promotional information is unavailable. Fourth, the methods we propose are fully automatic and easy to implement compared to Huang et al. (2014).

The remainder of the paper is organised as follows: section 2 summarizes previous studies in the literature related to forecasting retailer product sales and the change of the effect of marketing activities. Section 3 explains the origins and the consequence of the structural change problem. In section 4, we introduce two methods which are used in the macroeconomics area to deal with the structural change problem. Section 5 explores the data. In section 6, we propose our new three-stage forecasting methods. Section 7 describes the design of the model evaluation. Section 8 summarizes and discusses the evaluation results in order to provide a convincing demonstration of their performance. In Section 9, we explore the characteristics of the situations where the proposed models garner the greatest improvements. In the last section, we make recommendations for both manufacturers and retailers, address research limitations, and highlight directions for future research.

## Literature review

In practice, many retailers forecast their product sales at SKU level using a two-stage ‘base-lift’ method. The method entails dividing the data into promoted and non-promoted periods based on whether the focal SKU is being promoted. The method is a combination of simple univariate methods (for the non-promoted period) and human judgments by brand/category managers (for the promoted period) (Fildes, Goodwin, Lawrence, & Nikolopoulos, 2009; Fildes et al., 2008). A number of studies has been devoted to helping managers with better adjustment procedures by overcoming their cognitive biases (Lee, Goodwin, Fildes, Nikolopoulos, & Lawrence, 2007; Petropoulos, Fildes, & Goodwin, 2016). Other studies try to improve the adjustment with model-based forecasting systems. e.g., they estimate the ‘lift’ effect by the promotional event based on information related to previous promotions, store/category features, and manufacturers etc. (L. Cooper et al., 1999; L. G. Cooper & Giuffrida, 2000; Trusov et al., 2006). One limitation of these methods is that, as they split the data into two periods, they tend to overlook the information in the promoted period when forecasting the product sales in the non-promoted period, and vice versa. Other studies have proposed more holistic methods to generate the forecasts. Divakar et al. (2005) developed the CHAN4CAST system with models of dynamic regression structures to forecast brand volume sales for the manufacturer/channel. Gür Ali et al. (2009) evaluated the forecasting performance of support vector machine (SVM) models and regression tree models. Huang et al. (2014) proposed two-stage general-to-specific ADL models which incorporate competitive promotional information within the same product category of the focal product. Ma et al. (2016) further integrated the promotional information not only from the same category but also from other related categories.

The studies described above try to generate accurate forecasts by capturing the various effects of the marketing activities including prices and promotions. For example, previous studies suggest that price reductions and promotions increase the short term sales of the focal product (Blattberg, Briesch, & Fox, 1995). The price reductions and promotions not only increase the product sales at the focal period but also potentially reduce the sales before and after the focal period as customers may delay or stockpile their purchases (Mace & Neslin, 2004; Van Heerde, Gupta, & Wittink, 2003). The price and promotions also have competitive effects on the sales of other products within and across product categories. (R. L. Andrews, Currim, Leeflang, & Lim, 2008; Wedel & Zhang, 2004).

Evidence also shows that the effect of prices and promotions may change over time. For example, Wildt (1976) and Wildt and Winer (1983) suggest the effect of the marketing activities may change due to the change in economic conditions, consumer tastes, and the competition environment. Customers may find price reductions and promotions more attractive during the period of an economic crunch compared to other time periods. Mahajan, Bretschneider, and Bradford (1980) also found that the effect of prices and promotions change during the different stages of the product lifecycle. Meeran et al. (2017) found that customers have different tastes and preferences when they accumulate more knowledge of the product, when they seek variety, and when they reach a different social status and then decide to adopt a different lifestyle. These individual changes lead to substantial aggregate effects on the product sales. Previous studies found that the introduction of store-own brands in a product category decreases the promotional elasticities of premium national brands and increase promotional elasticities of the second tier national brands (Nijs, Dekimpe, Steenkamps, & Hanssens, 2001; Van Heerde, Srinivasan, & Dekimpe, 2008). However, current studies which forecast retailer product sales at SKU level all assume constant effect of the marketing activities. As a result, their methods may potetnially be subject to the problem of structural change, which we demonstrate in the next section.

## 3. Structural change and potential forecast bias

Conventional forecasting methods for retailer product sales assume constant parameters and overlook the change in the effect of the marketing activities. As a result, the forecasts generated by these methods may potentially be biased and less accurate (Allen & Fildes, 2001; Armstrong, 2001). This has been addressed by previous studies and is referred as the problem of structural change[[2]](#footnote-2) (e.g., Castle, Doornik, & Hendry, 2008; Hendry, 2018; H. M. Pesaran & Timmermann, 2007). H. M. Pesaran and Timmermann (2005) demonstrated analytically how a structural change lead to forecast bias using a simple regression model without an intercept. For example, we may denote that during the time periods of , the unobserved data generating process is:

(1)

where, and are the vectors of the dependent variable and independent variable respectively. is the vector of the error term. (where *i*=1,2) are the vectors of the parameter coefficients. is an indicator which equals to 1 before week (where ) and 0 afterwards. Therefore, we have a structural change where the true parameter of the independent variable changes from to after . We may estimate a model with a functional form congruent with the data generating process (e.g., ) based on the data before and after the structural change, e.g., ,. Thus, the OLS estimate of the parameter is:

(2)

where and are respectively the vectors of the independent variable and dependent variable for the time periods from week *m* to week *T*. We assume no structural change after week *T*. e.g., . Suppose that we are interested in the one-step ahead forecast. Thus, the one-step ahead error is:

(3)

where is the vector of the independent variable for the time periods from week *m* to . is the vector of error term for the time periods from week *m* to *T*. is the error term at week . Therefore, the forecast at week is biased as the expected value of the equation (3) is unequal to zero. e.g., . For more general cases where the model has the intercept term and endogenous explanatory variables, the forecast bias can be demonstrated using Monte Carlo simulation (M. P. Clements & Hendry, 1999; H. M. Pesaran & Timmermann, 2005, 2007)[[3]](#footnote-3).

In this study, we implement two methods to mitigate the problem of structural change. One is the Intercept Correction (IC) method which specifies non-zero values for the model’s errors in the forecasting period (Clark & McCracken, 2007; M. B. Clements & Hendry, 1994; M. P. Clements & Hendry, 1999). For example, if we can identify that the model is subject to structural changes, we can estimate the forecast bias by taking the average value of the most recent residuals, e.g., , where is the number of residuals. When , the estimated bias reduces to , which is the residual at the forecast origin (e.g., Chevillon, 2016). We can then add the estimated bias back to the out-of-sample forecasts. In the retailer context, product sales at SKU level often exhibit large variations, unexpected outliers, and missing values, which makes estimating the forecast bias a difficult task. Also, by adding the estimated bias back to the out-of-sample forecasts, we inevitably bear the cost of inflated forecasting error variance (see M. P. Clements & Hendry, 1999). An alternative method is to combine the forecasts which are generated by the same model but with different estimation windows, expecting a trade-off between the reduced forecast bias and potentially inflated forecast error variance. This method is referred as the estimation window combining (EWC) method (H. M. Pesaran & Timmermann, 2005; M. H. Pesaran & Pick, 2011; M. H. Pesaran, Schuermann, & Smith, 2009). We include the analytically demonstration of the EWC method in Appendix A. Overall, whether the IC method and the EWC method could lead to improved forecasting performance become empirical questions.

## The data

In this study, we evaluate the forecasting performance of various models using the retail dataset made available by the Information Resources, Inc. (IRI) company. A description of the dataset can be found in Bronnenberg, Kruger, and Mela (2008). The dataset contains weekly data at SKU level with variables including product unit sales, price, features, and displays etc. We initially conduct our evaluation based on 1831 SKU’s for 28 product categories from 28 different stores. Table 1 shows the basic statistics of the selected SKU’s during a period of 202 weeks for each product category[[4]](#footnote-4). Some product categories (e.g., Carbonated Beverages and Hotdog) exhibit much higher promotional intensity compared to others (e.g., Margarine/Butter and Mayonnaise). Figure 1 exhibits the data series for a typical SKU in the Beer category as an example: it indicates that sales spikes are usually associated with the price reductions and feature/display promotions of the focal product, as well as calendar events (e.g., Halloween, Thanksgiving, and Christmas etc.).

Table 1. Statistical description of different product categories

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Category | Price mean | Sales mean | Display percentage\* | Feature percentage\*\* | Number of SKU's |
| Beer | 8.3 | 20.6 | 13.90% | 4.00% | 169 |
| Blades | 8.1 | 14.6 | 7.40% | 2.20% | 22 |
| Carbonated Beverages | 2.1 | 113.6 | 26.80% | 15.60% | 82 |
| Cigarette | 22.3 | 22.2 | 0.00% | 0.80% | 203 |
| Coffee | 5.2 | 14.5 | 5.20% | 2.90% | 86 |
| Cold cereal | 3.5 | 70.7 | 4.00% | 18.10% | 125 |
| Deodorant | 2.7 | 6.9 | 4.10% | 5.20% | 126 |
| Face Tissue | 2.1 | 75.8 | 0.30% | 11.70% | 6 |
| Frozen Dinner | 2 | 43.8 | 5.30% | 23.70% | 87 |
| Frozen pizza | 3.4 | 31.2 | 8.90% | 9.10% | 147 |
| Household Cleaner | 2.5 | 29.9 | 0.30% | 3.60% | 18 |
| Hotdog | 4 | 68.6 | 13.20% | 15.60% | 35 |
| Laundry Detergent | 8.8 | 28.9 | 2.30% | 8.80% | 57 |
| Margarine/Butter | 2 | 71.4 | 0.10% | 6.30% | 36 |
| Mayonnaise | 3 | 79.7 | 3.00% | 0.40% | 22 |
| Milk | 2.5 | 222.3 | 2.10% | 1.80% | 30 |
| Mustard & Ketchup | 2.1 | 64.5 | 5.30% | 0.90% | 22 |
| Peanut butter | 3.7 | 34.2 | 3.20% | 0.60% | 15 |
| Photo | 7.2 | 9.2 | 4.60% | 5.10% | 13 |
| Salty snacks | 2.3 | 50.9 | 6.70% | 5.00% | 101 |
| Shampoo | 3.5 | 9.9 | 12.80% | 7.10% | 70 |
| Soup | 1.5 | 61.6 | 1.20% | 9.70% | 139 |
| Spaghetti sauce | 2.4 | 39.1 | 1.60% | 6.50% | 52 |
| Sugar substitutes | 2.8 | 14.5 | 0.10% | 1.40% | 20 |
| Toilet Tissue | 5.4 | 89.1 | 4.30% | 8.30% | 20 |
| Toothbrush | 2.6 | 8.7 | 3.10% | 6.30% | 28 |
| Toothpaste | 2.8 | 35.5 | 11.00% | 12.50% | 25 |
| Yogurt | 1.1 | 115.1 | 0.70% | 6.30% | 75 |

\* \*\*Display percentage and Feature percentage indicate the percentage of weeks during the 202-week time periods when the focal product is being promoted for Display Advertisement and Feature Display.

Figure 1. Store level data for an SKU in the Beer category[[5]](#footnote-5)



## Methodology

In this study, we propose new forecasting methods which take into account the problem of structural change. The new methods consist of three stages. At the first stage, we identify the most informative competitive explanatory variables for the focal product. Grocery retailers typically sell hundreds of SKU’s in a typical product category and this leads to hundreds of potential competitive explanatory variables for the focal product. Incorporating all the variables into the model would easily overfit the model and even make the estimation infeasible (Martin & Kolassa, 2009). Therefore, we initially select the most relevant variables using the Least Absolute Shrinkage and Selection Operator (LASSO) procedure (Tibshirani, 1996). Specifically, we construct the following model for each SKU:

(4)

where represents log product sales of the focal product at week *t.*  
 is the matrix for the explanatory variables including product prices, features, and displays of all the products in the same product category.

*u* represents the identically distributed error term.

represents the vector for the parameter coefficients.  
*N* is the total number of SKUs for the category.  
 is the shrinkage factor.

The LASSO procedure imposes a constraint to the sum of the absolute values of the models’ parameter coefficients. It removes less relevant explanatory variables by pushing their parameter coefficients towards zero. We control the model simplification process using the shrinkage factor based on 10-fold cross validation (Ma & Fildes, 2017; Ma et al., 2016)[[6]](#footnote-6).

At the second stage, we construct the General Autoregressive Distributive Lag (ADL) model (Huang et al. 2014). We initially include the variables retained by the LASSO procedure in the general ADL model. One limitation of the LASSO procedure is that it could misses important variables under the condition of high multicollinearity (Fan & Lv, 2008; Ma et al., 2016). In practice, retailers tend to promote relevant products at the same time. Therefore, we intentionally include the marketing variables of the focal product in the general ADL model since promotional variables of the focal variable are usually more important compared to the variables of other products (Bucklin, Gupta, & Siddarth, 1998). We include the dynamic effect of these variables as well as a time variable to capture the potential trend, 12 deterministic four-week dummy variables to capture seasonality, and other dummy variables to capture calendar events. We refer this model as the general ADL model:

(5)

where is the log sales of the focal product at week .

is the term which captures any potential trend during the estimation period (Song & Witt, 2003).

and represent the log price of the focal product and a competitive product, *m*, at week .

and represents the Feature dummy for the focal product at week .

is the four-week-dummy variable.  
 is the dummy variable for the calendar event at week . The dummy variable represents the week of the calendar event when , and the week before the event if . takes the values from 1 to 9 representing all the calendar events*[[7]](#footnote-7)*.

are the parameters.  
 is the error term and we assume .

is the order of the lags and is set as 2.

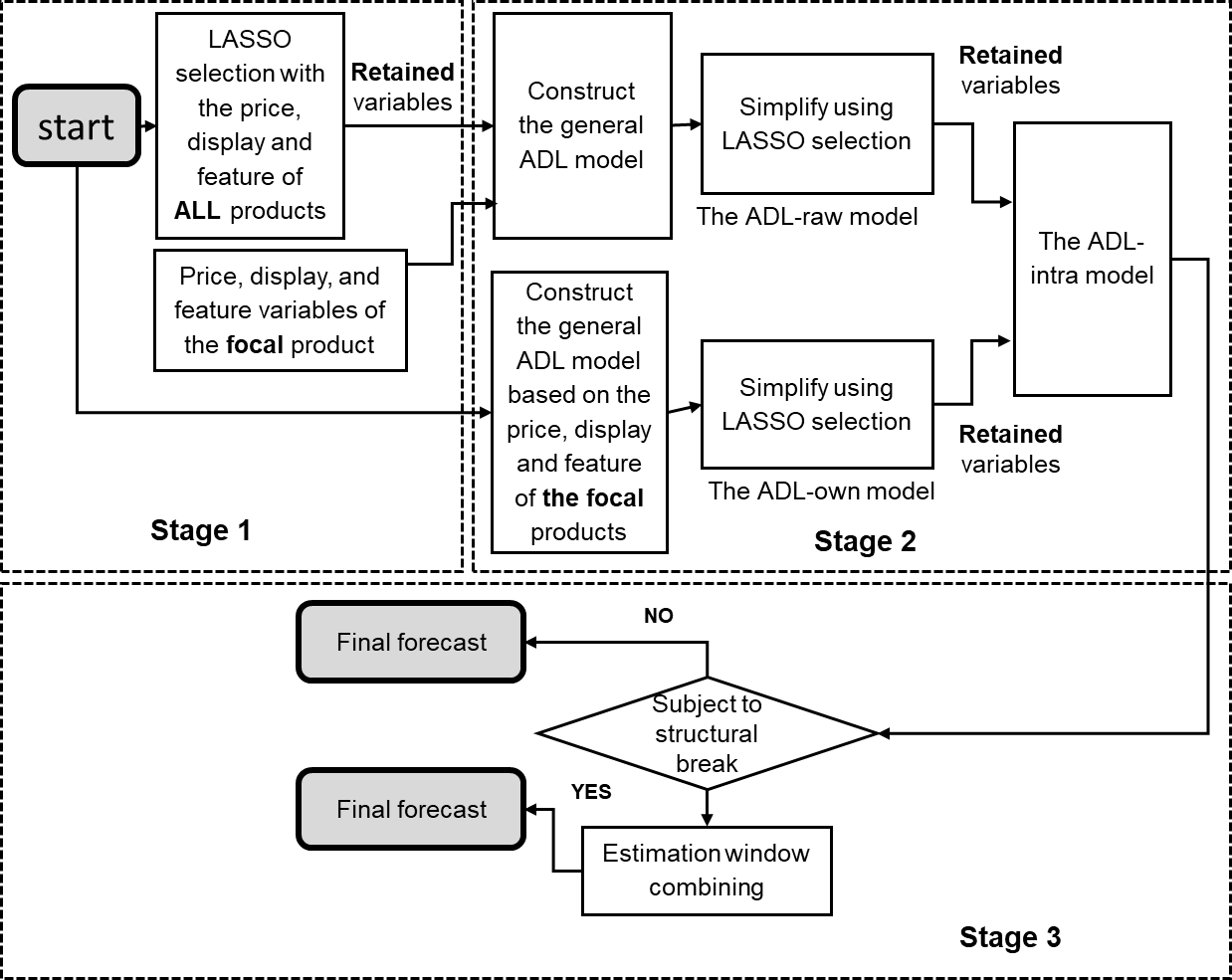
*, ,* and are the numbers of selected competitive price, Feature, and Display variables for the product category.

The specified general ADL model may have a large number of explanatory variables, and some of them may be correlated with others or do not convey any effective information. Therefore, we simplify the general ADL model by conducting the LASSO procedure (we refer to this simplified ADL model as the ADL-raw model thereafter). At this stage, we use the LASSO procedure as a model specification strategy rather than a variable selection method as previous studies indicate that models simplified by the LASSO procedure could have good forecasting performance and outperform traditional models specified based on statistical significance (Epprecht, Guegan, & Veiga, 2013; Ma et al., 2016). The LASSO procedure also enables the automation of the statistical forecasting task which becomes essential as typically grocery retailers stock a tremendous number of SKUs (L. Cooper et al., 1999). However, to mitigate the issue that the LASSO procedure could miss important variables under the condition of multicollinearity, we construct a supplementary parallel ADL model which has a similar specification compared to the ADL-intra model but only includes the price and promotion variables of the focal product:

(6)

We also simplify this model using the LASSO procedure (we refer to this simplified model as the ADL-own model thereafter). We then incorporate the marketing variables retained by the ADL-own model into the ADL-raw model (we refer to the resulted model as the ADL-intra model). In our modelling process, we initially use the LASSO procedure to select the important marketing variables and then to simplify the initial general model. The simplified model (i.e., the ADL-raw model) benefits from a parsimonious specification. On the other side, we also create “opportunities” to bring back the marketing variables of the focal product, which makes it less likely for the ADL-intra model to wrongfully miss the important variables, though at a cost of efficiency.

Figure 2. An illustration for the three-stages of the ADL-intra-EWC model



At the final stage, we integrate the ADL-intra model with the EWC method and the IC method respectively to account for the structural change problem. We implement the EWC method and the IC method to the ADL-intra model only if a sequential Chow test suggests the existence of structural change. Otherwise, we keep the forecasts generated by the ADL-intra model as the final forecasts.

For the sequential Chow test, we simply conduct the Chow test for up to 95% of the weeks in the estimation period. For example, with an estimation period of 160 weeks, we may initially conduct the Chow test assuming there is a structural change occurring at week 5 and we obtain the p-value. We then conduct the Chow test for week 6, 7, and so forth until week 156 and each time we obtain the p-value accordingly. We reserve at least 5% of the weeks to ensure there are enough observations to for the test[[8]](#footnote-8). Thus, for the sequential Chow test, we obtain up to 152 p-values. The null hypothesis of no structural change will be rejected only when none of these p-value is below the threshold. To mitigate the multiple comparison problem, we adopt a very small threshold, i.e., 0.001. Previous studies have proposed alternative tests which focus on estimating the number of the multiple structural changes and their locations, and are usually associated with stringent assumptions (e.g., Donald W K Andrews, 1993; Donald W. K. Andrews & Ploberger, 1994; Bai & Perron, 1998, 2003; Brown, Durbin, & Evans, 1975). In this study, we only need to investigate the existence of the structural change. Thus, we conduct the sequential Chow test which serves for this purpose and simple to implement. We refer to the final resulted models as the ADL-intra-EWC model and the ADL-intra-IC model respectively and we expect these models to generate more accurate forecasts by taking into account the structural change problem. Figure 2 provides a guide to implementing the ADL-intra-EWC model. In Figure 2, the ADL-intra-IC model can be implemented analogously by replacing the EWC method with the IC method once we confirm that the model is subject to structrual change.

## The experimental design

In this study, we consider the Base-lift method as the benchmark model. The method is widely used in practice and evaluated in previous studies(e.g., L. Cooper et al., 1999; Gür Ali et al., 2009; Huang et al., 2014; Ma et al., 2016). The forecasts for week *t* by this method can be described as follows:

where represents the initial baseline forecast for week by the simple exponential smoothing model. represents the actual sales of the focal product during the previous week given that the it was not promoted. is the parameter of the simple exponential smoothing model. It is estimated by minimizing the in-sample mean squared errors. The adjustment is therefore calculated as the increased sales of the focal product by its most recent promotion compared to the corresponding initial baseline.

We have the following candidate models: 1) The ADL-own model; 2) The ADL-intra model; 3) The ADL-intra-EWC model; 4) The ADL-own-EWC model: similar to the ADL-intra-EWC model except that the ADL-intra model is replaced by the ADL-own model at the final stage; 5) The ADL-intra-IC model; 6) The ADL-own-IC model: similar to the ADL-intra-IC model except that the ADL-intra model is replaced by the ADL-own model at the final stage. We implement the models using MODEL procedure with macros in SAS 9.4. The model parameters are estimated using the OLS estimator.

In this study, we evaluate the forecasting performance of these models with 18 rolling origins for robustness (Tashman, 2000). We specify the model with an estimation window of 160 weeks. For each rolling event, we move the estimation window two weeks forward and re-specify the model. We presume the value of the price and promotional information to be known, as it is part of the retailer’s inventory plan, and we use the forecast value of the product sales when the forecast horizon is beyond one week. We generate one to week-ahead forecasts, where is 1, 4, and 8, to approximate the situation retailers face in practice. For the EWC method, we engage ten estimation windows with different lengths (e.g., for the initial estimation period [1,160], we estimate the model with ten estimation windows including [1, 160], [3, 160], and so forth, until [19, 160]), and generate ten sets of forecasts accordingly). We combine the ten sets of forecasts with equal weights. For the IC methods, we estimate the forecast bias as the average value of the sixteen most recent residuals and add the value equally to the forecasts of all the forecast horizons.

We evaluate the models with various error measures which approximate the loss function of the retailer from different aspects. We include traditional error measures including the Mean Absolute Error (*MAE*), the symmetric Mean Absolute Percentage Error (*sMAPE*) and the scaled Mean Squared Error (sMSE). We also include recently developed error measures including the Mean Absolute Scaled Error (*MASE*) developed by Hyndman and Koehler (2006) and the Relative Average Mean Absolute Error (*RelAvgMAE*) developed by Davydenko and Fildes (2013). The two latter error measures for SKUs based on a forecast horizon of 1 to (e.g., and =1, 4 and 8) are as follows:

, where

Where

Where and are the MASE and the AvgRelMAE based on one to *H* forecast horizon (=1, 4 and 8). and are respectively the *h*-step ahead actual value and forecast value for data series based on the rolling event. There are *S* data series and *K* rolling events (*S*= 1831 and *K*=18). is the total number of observations in the estimation window (i.e., ). Before we transform the log values to levels for evaluation, we adjust the final forecasts by adding one-half mean squared error, which mitigate the bias caused by the logarithm transformation (e.g., L. Cooper et al., 1999; Ma et al., 2016).

## Results and discussion

In Table 2, we summarize the forecasting performance of the models across all the product categories. Table 3 shows the results of the Diebold-Mariana (DM) test for the statistical significance of the difference between the models’ forecasting performance. (Diebold & Mariano, 1995; Harvey, Leybourne, & Newbold, 1997)[[9]](#footnote-9). We find the following from the analysis of the comparisons of forecasts from the different models:

1. The Base-lift model generates the least accurate forecasts.
2. The ADL-intra model outperforms the ADL-own model, which is consistent with the findings in Huang et al. (2014).
3. The ADL-own-EWC model outperforms the ADL-own model for all the error measures.
4. The ADL-own-IC model generally outperforms the ADL-own model except for the *MAE* which is scale dependent.
5. The ADL-intra-EWC model outperforms the ADL-intra model for all the error measures.
6. The ADL-intra-IC model generally outperforms the ADL-intra model except for the *MAE* and the *MSE* error measures for longer forecast horizons (e.g., *h*=4 and 8).

Overall, The ADL-intra-EWC model and the ADL-intra-IC model generate the most accurate forecasts. In Table 6, we compare the forecasting performance of the ADL-intra-EWC model and the ADL-inter-IC model to the ADL-intra model for each individual product category based on the MASE for one to eight-week forecast horizon. We focus on the ADL-intra-EWC model and the ADL-inter-IC model because they have the best forecasting performance overall and the ADL-intra model has a similar model specification expect that it overlooks the issue of structural change. The comparison results for other error measures and horizons are similar. The ADL-intra-EWC model and the ADL-intra-IC model outperforms the ADL-intra model for the majorities of product categories (e.g., 20 and 17 respectively, out of 28 categories). They do not outperform the ADL-intra model for all product categories due to the heterogeneity of the data characteristics across different product categories (e.g., Ma et al., 2016).

Table 2. The forecasting performance of the models for all forecast period

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Forecast horizon is 1 to 8 weeks ahead, for all forecast period | | | | | | | | | | |
| Model/measure | MAE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank | Scaled MSE | Rank |
| Base-lift | 22.919 | 7 | 46.98% | 7 | 0.775311 | 7 | 1.1444 | 7 | 0.2234 | 7 |
| ADL-own | 15.755 | 5 | 40.81% | 6 | 0.697303 | 6 | 1.0000 | 6 | 0.1575 | 5 |
| ADL-intra | 15.436 | 2 | 40.51% | 3 | 0.695222 | 4 | 0.9941 | 3 | 0.1553 | 2 |
| ADL-own-EWC | 15.673 | 4 | 40.68% | 4 | 0.695964 | 5 | 0.9956 | 4 | 0.1570 | 4 |
| ADL-own-IC | 16.233 | 6 | 40.76% | 5 | 0.694034 | 3 | 0.9992 | 5 | 0.1596 | 6 |
| ADL-intra-EWC | 15.354 | 1 | 40.41% | 1 | 0.693915 | 2 | 0.9905 | 1 | 0.1548 | 1 |
| ADL-intra-IC | 15.595 | 3 | 40.46% | 2 | 0.692854 | 1 | 0.9936 | 2 | 0.1568 | 3 |
| Forecast horizon is 1 to 4 weeks ahead, for all forecast period | | | | | | | | | | |
| Model/measure | MAE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank | Scaled MSE | Rank |
| Base-lift | 22.669 | 7 | 46.24% | 7 | 0.761699 | 7 | 1.1365 | 7 | 0.2186 | 7 |
| ADL-own | 15.630 | 5 | 40.45% | 6 | 0.690272 | 6 | 1.0000 | 6 | 0.1548 | 5 |
| ADL-intra | 15.157 | 2 | 40.12% | 3 | 0.686329 | 4 | 0.9913 | 3 | 0.1514 | 2 |
| ADL-own-EWC | 15.546 | 4 | 40.31% | 5 | 0.688358 | 5 | 0.9950 | 5 | 0.1540 | 4 |
| ADL-own-IC | 15.942 | 6 | 40.25% | 4 | 0.683757 | 2 | 0.9948 | 4 | 0.1553 | 6 |
| ADL-intra-EWC | 15.089 | 1 | 40.01% | 2 | 0.684993 | 3 | 0.9876 | 2 | 0.1509 | 1 |
| ADL-intra-IC | 15.211 | 3 | 39.93% | 1 | 0.681286 | 1 | 0.9871 | 1 | 0.1517 | 3 |
| Forecast horizon is 1 week ahead, for all forecast period | | | | | | | | | | |
| Model/measure | MAE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank | Scaled MSE | Rank |
| Base-lift | 24.990 | 7 | 45.415% | 7 | 0.762 | 7 | 1.1279 | 7 | 0.2261 | 7 |
| ADL-own | 16.662 | 5 | 39.873% | 6 | 0.689 | 6 | 1.0000 | 6 | 0.1561 | 6 |
| ADL-intra | 15.661 | 3 | 39.434% | 3 | 0.686 | 4 | 0.9883 | 3 | 0.1529 | 3 |
| ADL-own-EWC | 16.588 | 4 | 39.720% | 5 | 0.686 | 5 | 0.9955 | 5 | 0.1549 | 4 |
| ADL-own-IC | 17.015 | 6 | 39.519% | 4 | 0.680 | 2 | 0.9902 | 4 | 0.1552 | 5 |
| ADL-intra-EWC | 15.595 | 1 | 39.329% | 2 | 0.684 | 3 | 0.9850 | 2 | 0.1523 | 2 |
| ADL-intra-IC | 15.653 | 2 | 39.148% | 1 | 0.679 | 1 | 0.9804 | 1 | 0.1520 | 1 |

Table 3. The results of the Diebold-Mariana (DM) test

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model 1 | Model 2 | MAE | | | SMAPE | | | MASE | | | scaled MSE | | |
| *H*=1 | *H*=1 to 4 | *H*=1 to 8 | *H*=1 | *H*=1 to 4 | *H*=1 to 8 | *H*=1 | *H*=1 to 4 | *H*=1 to 8 | *H*=1 | *H*=1 to 4 | *H*=1 to 8 |
| ADL-own | Base-lift | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| ADL-own | ADL-intra | 0.000 | 0.001 | 0.015 | 0.000 | 0.000 | 0.000 | 0.233 | 0.026 | 0.157 | 0.443 | 0.380 | 0.453 |
| ADL-own | ADL-own-EWC | 0.106 | 0.005 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.104 | 0.294 | 0.148 | 0.335 | 0.258 |
| ADL-own | ADL-own-IC | 0.064 | 0.008 | 0.000 | 0.000 | 0.000 | 0.259 | 0.000 | 0.000 | 0.009 | 0.388 | 0.138 | 0.001 |
| ADL-intra | ADL-intra-EWC | 0.138 | 0.013 | 0.002 | 0.000 | 0.000 | 0.000 | 0.005 | 0.124 | 0.100 | 0.652 | 0.259 | 0.308 |
| ADL-intra | ADL-intra-IC | 0.946 | 0.469 | 0.021 | 0.000 | 0.000 | 0.277 | 0.000 | 0.000 | 0.030 | 0.169 | 0.011 | 0.001 |

We also investigate the models’ forecasting performance for the time periods depending on whether the focal product is being promoted because the corresponding sales tend to exhibit very different levels of variations[[10]](#footnote-10). We refer these two periods as the promoted period and non-promoted period respectively. Table 4 shows the forecasting performance of the models for the promoted period and the non-promoted forecast period respectively for one to eight-week forecast horizon[[11]](#footnote-11). The results are similar compared to those in Table 2. Of the many detailed comparisons possible, the following seem particularly important: the ADL-intra-IC model has the best forecasting performance for the non-promoted period but only has moderate performance for the promoted period. A possible explanation is that the estimated bias used for the correction gets submerged by the high variations of the product sales when the focal product is being promoted. In contrast, the ADL-intra-EWC model has the best performance for the promoted period. Therefore, we forge an exploratory combined model between these two models, named as the ADL-EWC-IC model. The ADL-EWC-IC model will be identical to the ADL-intra-EWC model for the promoted period and to the ADL-intra-IC model for the non-promoted period. To make a fair comparison, we evaluate the performance of the ADL-EWC-IC model based on previously unseen data (e.g., the data from the same 28 product categories but from a set of different 28 stores). Table 5 shows the forecasting performance of the ADL-EWC-IC model compared to other three models[[12]](#footnote-12). The results indicate that the ADL-EWC-IC model generally generates the most accurate forecasts across all the models even for previously unseen data.

Table 4.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Forecast horizon is 1 to 8 weeks ahead, for the promoted period | | | | | | | | | | |
| Model/measure | MAE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank | scaled MSE | Rank |
| Base-lift | 119.330 | 7 | 87.26% | 7 | 1.915 | 7 | 1.3705 | 7 | 2.4742 | 7 |
| ADL-own | 65.272 | 5 | 47.56% | 5 | 1.329 | 5 | 1.0000 | 4 | 1.0719 | 5 |
| ADL-intra | 63.100 | 2 | 46.04% | 2 | 1.307 | 2 | 0.9795 | 2 | 1.0265 | 2 |
| ADL-own-EWC | 65.010 | 3 | 47.43% | 4 | 1.325 | 3 | 0.9955 | 3 | 1.0662 | 4 |
| ADL-own-IC | 69.677 | 6 | 47.95% | 6 | 1.354 | 6 | 1.0208 | 6 | 1.1299 | 6 |
| ADL-intra-EWC | 62.737 | 1 | 45.91% | 1 | 1.303 | 1 | 0.9756 | 1 | 1.0196 | 1 |
| ADL-intra-IC | 65.013 | 4 | 46.30% | 3 | 1.327 | 4 | 1.0035 | 5 | 1.0651 | 3 |
| Forecast horizon is 1 to 8 weeks ahead, for the non-promoted period | | | | | | | | | | |
| Model/measure | MAE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank | scaled MSE | Rank |
| Base-lift | 8.837 | 7 | 41.10% | 7 | 0.609 | 7 | 1.0083 | 7 | 0.0973 | 7 |
| ADL-own | 8.523 | 6 | 39.83% | 6 | 0.605 | 5 | 1.0000 | 6 | 0.0921 | 5 |
| ADL-intra | 8.475 | 5 | 39.70% | 4 | 0.606 | 6 | 0.9986 | 4 | 0.0922 | 6 |
| ADL-own-EWC | 8.467 | 4 | 39.70% | 3 | 0.604 | 3 | 0.9963 | 1 | 0.0920 | 3 |
| ADL-own-IC | 8.427 | 2 | 39.71% | 5 | 0.598 | 1 | 0.9995 | 5 | 0.0916 | 1 |
| ADL-intra-EWC | 8.433 | 3 | 39.61% | 2 | 0.605 | 4 | 0.9964 | 2 | 0.0921 | 4 |
| ADL-intra-IC | 8.377 | 1 | 39.61% | 1 | 0.600 | 2 | 0.9976 | 3 | 0.0918 | 2 |

Table 5. The forecast results based on previously unseen data from a different set of 28 stores.

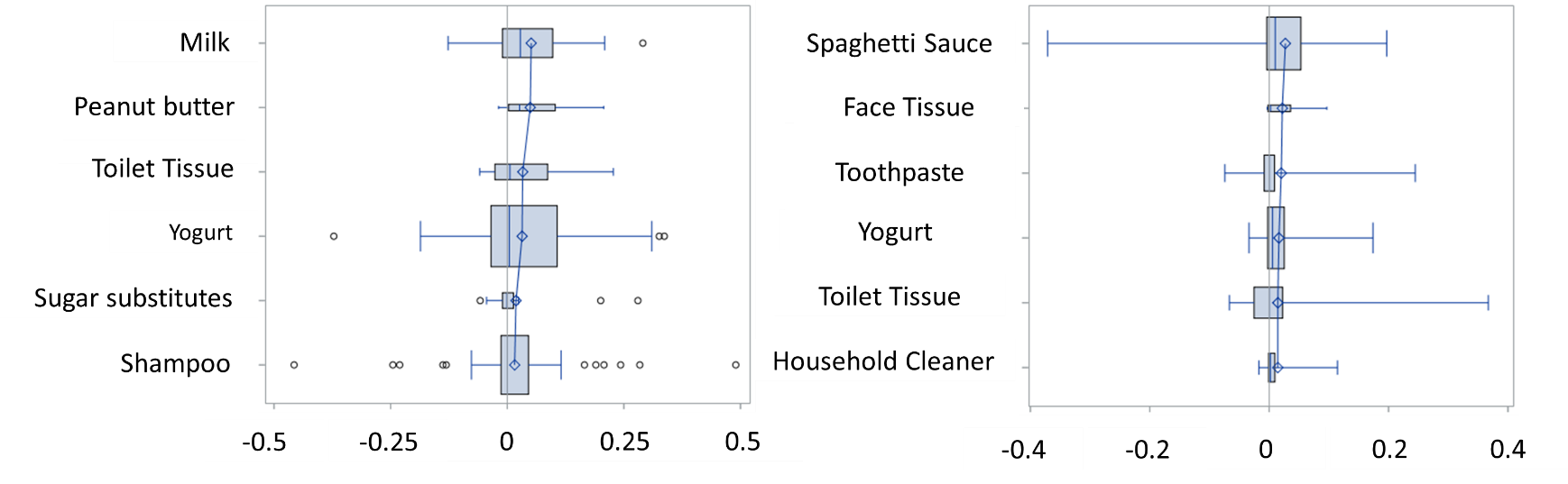
|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| All forecast period, for 1 to 8 weeks ahead | | | | | | | | | | |
| Model/measure | MAE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank | scaled MSE | Rank |
| ADL-intra | 13.441 | 3 | 40.01% | 4 | 0.770 | 4 | 1.0000 | 4 | 0.1689 | 3 |
| ADL-intra-EWC | 13.473 | 4 | 39.89% | 3 | 0.769 | 3 | 0.9964 | 3 | 0.1690 | 4 |
| ADL-intra-IC | 13.339 | 1 | 39.60% | 2 | 0.762 | 2 | 0.9885 | 2 | 0.1674 | 1 |
| ADL-EWC-IC | 13.387 | 2 | 39.59% | 1 | 0.762 | 1 | 0.9876 | 1 | 0.1677 | 2 |
| promoted period, for 1 to 8 weeks ahead | | | | | | | | | | |
| Model/measure | MAE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank | scaled MSE | Rank |
| ADL-intra | 55.110 | 1 | 45.96% | 3 | 1.569417 | 4 | 1.0000 | 3 | 1.2509 | 2 |
| ADL-intra-EWC | 55.549 | 3 | 45.90% | 1 | 1.568883 | 1 | 0.9960 | 1 | 1.2549 | 3 |
| ADL-intra-IC | 55.112 | 2 | 45.99% | 4 | 1.569142 | 3 | 1.0090 | 4 | 1.2477 | 1 |
| ADL-EWC-IC | 55.549 | 3 | 45.90% | 1 | 1.568883 | 1 | 0.9960 | 1 | 1.2549 | 3 |
| non-promoted period, for 1 to 8 weeks ahead | | | | | | | | | | |
| Model/measure | MAE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank | scaled MSE | Rank |
| ADL-intra | 8.296 | 4 | 39.27% | 4 | 0.67148 | 4 | 1.0000 | 4 | 0.1047 | 4 |
| ADL-intra-EWC | 8.279 | 3 | 39.15% | 3 | 0.670104 | 3 | 0.9963 | 3 | 0.1047 | 3 |
| ADL-intra-IC | 8.182 | 1 | 38.81% | 1 | 0.66279 | 1 | 0.9871 | 1 | 0.1036 | 1 |
| ADL-EWC-IC | 8.182 | 1 | 38.81% | 1 | 0.66279 | 1 | 0.9871 | 1 | 0.1036 | 1 |

In Table 6, we compare the forecasting performance of the ADL-intra-EWC model and the ADL-inter-IC model to the ADL-intra model for each individual product category based on the MASE for one to eight-week forecast horizon. We focus on the ADL-intra-EWC model and the ADL-inter-IC model because they have the best forecasting performance overall and the ADL-intra model has a similar model specification but overlooks the issue of structural change. The comparison results for other error measures and horizons are similar. The ADL-intra-EWC model and the ADL-intra-IC model outperforms the ADL-intra model for the majorities of the product categories (e.g., 20 and 17 respectively, out of 28 categories). They do not outperform the ADL-intra model for all product categories due to the heterogeneity of the data characteristics across different product categories (e.g., Ma et al., 2016). Figure 3 show further details using boxplots for their best performing product categories. In the figures, positive values indicate the percentage reduction of the MASE by the ADL-intra-EWC model and by the ADL-intra-IC model compared to the ADL-intra model.

Table 6. The percentage reduction by the ADL-intra-EWC model and the ADL-intra-IC model compared to the ADL-intra model for each product category for the MASE for one to eight-week forecast horizon

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Category/MASE | ADL-intra-EWC | ADL-intra-IC | Category/MASE | ADL-intra-EWC | ADL-intra-IC |
| Beer | 0.12% | -0.58% | Mayonnaise | 0.07% | 0.58% |
| Blades | 0.20% | 2.19% | Milk | 1.04% | 6.25% |
| Carbonated Beverages | 0.40% | 0.10% | Mustard & Ketchup | 0.64% | -1.04% |
| Cigarette | 0.17% | 1.29% | Peanut butter | -0.15% | 5.11% |
| Coffee | -0.01% | 0.38% | Photo | 1.16% | 0.20% |
| Cold Cereal | 0.11% | -2.29% | Salty snacks | 0.02% | 0.25% |
| Deodorant | -0.01% | 1.74% | Shampoo | 0.38% | 1.56% |
| Face Tissue | 1.80% | -0.47% | Soup | 1.03% | -3.29% |
| Frozen Dinner | -0.67% | -0.70% | Spaghetti sauce | 1.61% | 1.67% |
| Frozen pizza | -1.71% | -1.73% | Sugar substitutes | 0.39% | 3.41% |
| Household Cleaner | 1.25% | 0.72% | Toilet Tissue | 0.04% | 2.45% |
| Hotdog | -0.44% | -4.05% | Toothbrush | -0.02% | -2.12% |
| Laundry Detergent | 0.43% | 0.62% | Toothpaste | 1.66% | -1.80% |
| Margarine/Butter | -0.57% | -0.76% | Yogurt | 1.78% | 4.47% |

Figure 3. Compare three models with the ADL-intra model for six product categories: results at SKU level, for the MASE, and for one to eight-week forecast horizon. (positive numbers indicate higher performance of the prosposed models)



The box widths are proportionate to the number of SKU’s for each product category. The square symbols, which are joined by lines for illustration, indicates the group means.

1. the ADL-intra-EWC model (b) the ADL-intra-IC model,

## Exploring the determinants of the forecasting improvement

The results in Table 6 show that our proposed models generate more accurate forecasts especially for some product categories (e.g., Yogurt, Milk, Toilet Tissue etc.). We further explore the determinants of the improvement of the forecasting performance of our proposed models at SKU level. This provides insights into for what types of SKUs we may get most benefit by using the proposed models. We consider the following data characteristics as potential determinants: 1) basic statistical measures for both the prices and sales variables including the average, standard deviation, skewness, range, kurtosis, and coefficient of variation; 2) the frequency of the feature and display promotions for each SKU; 3) more advanced statistical measures suggested by Fildes (1992). For example, we include the proportion of outliers for the sales of each SKU. The value of the sales for product *i* will be identified as an outlier if or , where is the differenced value of the sales for product *i*. and are the first and third quantiles of . We also include the randomness measure by regressing on , where is the sales value for product *i* at week *t* given that the outliers are removed and *T* is the time trend. The fitness of this autoregressive model (e.g., the R square) approximates the systematic variation in the sales data which could be captured by simple models. Lastly, we include the linear trend of product sales measured as the absolute value of the correlation between and the time trend. We then construct five orthogonal factors to represent the information originally contained in the fourteen explanatory variables described above, which mitigates the issue of multicollinearity[[13]](#footnote-13). Table 6 shows the correlation between the original fourteen explanatory variables and the constructed factors[[14]](#footnote-14). We may interpret factor 1 as “Outliers and general variations”, factor 2 as “Sales level and variation”, factor 3 as “Central tendency of sales”, factor 4 as “Price level and variation”, and factor 5 as “Randomness and growth”.

Thus, we can develop regression models to explore potential determinants of the forecasting improvement by the proposed models. Specifically, we construct four dependent variables which are the percentage reductions of the MASE by the ADL-intra-EWC model and the ADL-intra-IC model compared to the ADL-intra model, and the percentage reductions of the MASE by the ADL-own-EWC model and the ADL-own-IC model compared to the ADL-intra model. For robustness, we develop regression models with and without dummy variables for each product category.

Table 6. The pattern of the factors

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Variable | Factor1 | Factor2 | Factor3 | Factor4 | Factor5 |
| Proportion of outliers | 0.855 |  |  |  |  |
| Coefficient of variation (price) | 0.759 |  |  |  |  |
| Coefficient of variation (sales) | 0.714 |  |  |  |  |
| Frequency of Feature | 0.703 |  |  |  |  |
| Standard deviation of sales |  | 0.964 |  |  |  |
| Range of sales |  | 0.929 |  |  |  |
| Average sales |  | 0.807 |  |  |  |
| Frequency of Display |  | 0.281 |  |  |  |
| Kurtosis of sales |  |  | 0.973 |  |  |
| Skewness of sales |  |  | 0.881 |  |  |
| Standard deviation of price |  |  |  | 0.987 |  |
| Average price |  |  |  | 0.831 |  |
| Randomness |  |  |  |  | 0.992 |
| Trend |  |  |  |  | 0.703 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Table 7 The determinants of reductions of the MASE for one to eight weeks ahead horizon\*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Horizon = 1 to 8 weeks ahead | ADL-intra-EWC | | ADL-own-EWC | | ADL-intra-IC | | ADL-own-IC | |
| Parameter/estimate and p-values | Estimate | P-value | Estimate | P-value | Estimate | P-value | Estimate | P-value |
| Outliers and promotional variations | 0.07 | 0.434 | 0.11 | 0.303 | -1.09 | 0.000 | -1.45 | 0.000 |
| Sales level and variation | 0.12 | 0.173 | 0.16 | 0.105 | -0.21 | 0.340 | -0.93 | 0.000 |
| Central tendency of sales | -0.06 | 0.460 | -0.07 | 0.511 | -0.68 | 0.002 | -0.84 | 0.001 |
| Price level and variation | -0.12 | 0.149 | -0.17 | 0.092 | 0.07 | 0.742 | -0.09 | 0.721 |
| Randomness and growth | 0.38 | 0.000 | 0.45 | 0.000 | 0.63 | 0.004 | 0.80 | 0.001 |
| Intercept | 0.30 | 0.001 | 0.37 | 0.000 | -0.38 | 0.082 | -0.46 | 0.060 |
| Horizon = 1 to 8 weeks ahead | ADL-intra-EWC | | ADL-own-EWC | | ADL-intra-IC | | ADL-own-IC | |
| Parameter/estimate and p-values | Estimate | P-value | Estimate | P-value | Estimate | P-value | Estimate | P-value |
| Outliers and promotional variations | 0.21 | 0.119 | 0.41 | 0.009 | -0.45 | 0.000 | -0.60 | 0.000 |
| Sales level and variation | 0.12 | 0.172 | 0.20 | 0.055 | -0.12 | 0.595 | -0.85 | 0.001 |
| Central tendency of sales | -0.04 | 0.662 | 0.03 | 0.804 | -0.45 | 0.061 | -0.55 | 0.041 |
| Price level and variation | -0.12 | 0.338 | -0.30 | 0.046 | -0.10 | 0.761 | -0.39 | 0.284 |
| Randomness and growth | 0.32 | 0.000 | 0.38 | 0.000 | 0.48 | 0.039 | 0.56 | 0.033 |
| Intercept | 1.48 | 0.001 | 1.64 | 0.001 | 2.40 | 0.031 | 4.06 | 0.001 |

Table 7 reports the estimated parameters of the regression models for the MASE for the one to eight weeks ahead horizon[[15]](#footnote-15). The top half of Table 7 shows the results for the model without category dummy variables. For example, for the percentage reduction of the MASE by the ADL-intra-EWC model and by the ADL-intra-IC model compared to the ADL-intra model, the estimates for “Randomness and growth” are positive (e.g., 0.38 and 0.63) and statistically significant (e.g., a p-values smaller than 0.001, displayed as “0.000”, and 0.004). This indicates that, using the ADL-intra-EWC model and the ADL-intra-IC model lead to higher percentage reductions of the MASE for the SKU’s with higher randomness and trend (e.g., being difficult to forecast and exhibit a trend in sales), possibly because the SKUs of this type are more heavily associated with the structural change problem and forecast bias. The results also show that the ADL-intra-IC model and the ADL-own-IC model tend to have less advantages compared to the ADL-intra model and the ADL-own model respectively for the SKUs with a higher proportion of outliers and higher variations, possibly because that the ‘intercept correction’ for the bias can be submerged by high sales spikes which are usually ‘outliers’ and caused by promotions and it is more challenging to estimate the forecast bias under higher sales variations. Overall, the results here may indicate a possibility of determining the optimal sales forecasting method specifically for an SKU. However, the findings are only exploratory, and we leave it to future research.

## Conclusions, limitations and future research

Grocery retailers need to effectively manage their inventory and, to achieve that, they rely on effective forecasting models and welcome new approaches that will enable them to improve their current practices. Previous studies focus on incorporating additional information (e.g., Gür Ali et al., 2009; Huang et al., 2014; Ma et al., 2016). However, they all assume that the effect of the marketing activities such as price reductions and feature and display promotions remain unchanged over time. This assumption may not hold because of the impact of external factors including the change in economic conditions, the change in consumer taste, and new competition entry etc. The data on these factors are not always available, or, we do not actually know which of these external factors are causing the structural change. As a result, conventional models may be subject to the problem of structural change and potentially generate biased and less accurate forecasts.

Table 8. The percentage reductions for different error measures

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | MAE | SMAPE | MASE | AvgRelMAE | scaled MSE |
| ADL-own-EWC | -31.6% | -13.4% | -10.2% | -13.0% | -29.7% |
| ADL-own-IC | -29.2% | -13.3% | -10.5% | -12.7% | -28.6% |
| ADL-intra-EWC | -33.0% | -14.0% | -10.5% | -13.4% | -30.7% |
| ADL-intra-IC | -32.0% | -13.9% | -10.6% | -13.2% | -29.8% |

Our research focuses on how to mitigate the problem based on the data of marketing activities which retailers typically have control over. We propose models which take into account the potential forecast bias caused by structural change. The ADL-intra-EWC model generates forecasts which are the combination of various sets of forecasts by the ADL-intra model with different estimation windows under a condition when structural changes are detected. It tries to achieve an effective trade-off between the forecast bias and the forecast error variance. The ADL-intra-IC model tries to offset the potential forecast bias by adding the estimated forecast bias back to the error term at a cost of inflated forecast error variance when structural changes are detected. In the retailer context, the data at SKU level exhibit very different characteristics across different product categories and usually exhibit high levels of variations. Based on our empirical results, we find that these models outperform the ADL-intra model across all the 28 product categories. Table 8 shows the percentage reductions of various error measures by the ADL-intra-EWC model and the ADL-intra-IC model for one to eight-week forecast horizon[[16]](#footnote-16). Specifically, by using the ADL-intra-EWC model we can reduce the MASE by 10.6% compared to the current practice Base-lift method. Therefore, our study provides retailers more effective forecasting methods.

In this study, we have also evaluated the forecasting performance of the ADL-own-EWC model and the ADL-own-IC model. These methods are especially valuable to manufacturers since, under certain circumstances, competitive promotional information may not be available (M. Ali & Boylan, 2011; M. M. Ali, Babai, Boylan, & Syntetos, 2017). In our study, the ADL-own -EWC model and the ADL-own -IC model both outperform the ADL-own model across all the product categories. Table 8 also shows the percentage reductions of various error measures by the ADL-own-EWC model and the ADL-own-IC model for one to eight-week forecast horizon. Therefore, our study provides also manufacturers more effective forecasting methods.

In our study, the ADL-intra-EWC model has the best performance for the promoted forecast period while the ADL-intra-IC model dominates the non-promoted forecast period. We, therefore, forge an exploratory model combining the ADL-intra-EWC model and the ADL-intra-IC model based on if the focal product is being promoted. The resulted ADL-EWC-IC model thus generates the most accurate forecasts across all the candidate models for the original data and even for previously unseen data from another set of 28 stores.

We also explore the relationship between the relative advantage of the proposed models and the data characteristics of the product SKU. We find that the ADL-intra-EWC model and the ADL-own-EWC model tend to have better forecasting performances compared to the ADL-intra model and the ADL-own model respectively for the SKU’s with high randomness and trend, while the ADL-intra-IC model and the ADL-own-IC model tend to have more advantages compared to the ADL-intra model and the ADL-own model respectively for the SKU’s with high randomness and trend, with a low proportion of outliers and low level of general variations, and with a low level of sales central tendency.

The approach that we propose in this study is new to the area of retailer product sales forecasting, but we have also identified some areas where we feel further improvements could be beneficial. For example, there are studies which use splines smoothing method to model seasonality, which were found useful for electricity data (Nagbe, Cugliari, & Jacques, 2018). For the EWC method, we combine five sets of forecasts based on ten different estimation windows using equal weights. The forecasting performance may potentially be improved by changing the number of the estimation windows, by changing the minimum length of the estimation windows, and by exploring alternative forecasting combination schemes (e.g., based on k-fold evaluation). For the IC method, Clements and Hendry (1999) summarize various correction schemes each of which may have different effects on the trade-off between the bias and the error variance[[17]](#footnote-17). Furthermore, Ma et al. (2016) propose models which integrate both the intra- and the inter-category promotional information. Thus, we may further investigate how we can improve the forecasting performance with both the intra- and the inter-category promotional information while taking into account the structural change problem. A method alternative to the ADL-intra-EWC method and the ADL-intra-IC method is to directly model the changing process of the effect of the marketing activities. For example, the time-varying parameter model. However, a disadvantage of this method is that we need to make very strong assumptions of how the effect of the marketing activities change overtime. e.g., Foekens, Leeflang, and Wittink (1999) modelled the effect of the marketing activities as a linear function of previous promotional activities. The model has a sophisticated structure and was not developed for forecasting. Therefore, we leave the exploration of the potential of this type of model to future research. assumed that the effect of the marketing activities is a linear function of previous promotional activities. In summary, the models we proposed in this study produce consistently accurate forecasts. They also suffice the practical requirements of retail forecasting in that they are intuitive, they can be developed and operated automatically and also use readily available data on marketing activities.

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**Appendix A**

H. M. Pesaran and Timmermann (2007) show analytically, based on a simple model, that The Estimation Window Combining (EWC) method may potentially improve the forecasting accuracy by taking a trade-off between the reduced forecast bias and the inflated forecasting error variance. For example, if we know the location of the structural change (e.g., ), we can estimate the model exclusively with the post-break data, i.e., , and generate unbiased forecasts. As the location of the structural change is unknown, we may estimate the model using the data which are close to the forecast origin (e.g., we may keep *m* as large as possible) in conformity with maintaining the degrees of freedom so that that there are enough observations to estimate the model. If *m* by chance becomes larger than , the model will be exclusively estimated with the post-break data and will generate unbiased forecasts. However, this does not necessarily lead to higher forecasting accuracy because the corresponding forecasting error variance may increase due to a smaller estimation window (e.g., we are using less information to estimate the model). Thus, the Mean Squared Error (MSE) for week is , where

, , *μ*, and . can be interpreted as the squared forecast bias, and can be interpreted as the efficiency term ( is the forecasting error variance). Thus, the change of the MSE when we estimate the model with data compared to with the data is:

*(4)*

where is the MSE at week based on the estimation window [*m+*1*, T*]*.* When the observation at week *m* is excluded in the estimation, the change of the squared bias term (e.g., ) will always be non-positive (i.e., the bias will decrease), but the change of the efficiency term (e.g., ) depends on the error variance before and after the structural change. If (e.g., there are more pre-break variations compared to post-break variations in the error term which cannot be explained by the included explanatory variable), will be smaller than or equal to , and the MSE will decrease as the change for both the squared bias term and the efficiency term are non-positive. However, if , will be larger or equal to . Under this condition, the MSE may either increase or decrease depending on how the non-positive change of the squared bias term compares to the non-negative change of the efficiency term. As a result, when we exclude pre-break data and adopt a smaller estimation window, we may have either better or worse forecasting performance depending on the trade-off between the reduced forecast bias and the potentially inflated forecasting error variance. Therefore, the forecasts generated by the model with larger estimation windows may be subject to larger bias (contains more pre-break data) but associated with smaller forecast error variance (with more observations), and vice versa. As it is difficult to find the location of the structural change, we can combine the forecasts generated by the same model with different estimation windows, which may potentially lead to higher forecasting accuracy by making an effective trade-off between the forecast bias and the forecasting error variance (Clemen, 1989; Jose & Winkler, 2008).

Specifically, we may combine the forecasts with equal weights as it has been found effective and easy to implement.(M. Clements & Hendry, 1998; Dekker, van Donselaar, & Ouwehand, 2004; Fildes & Stekler, 2002; M. H. Pesaran et al., 2009). We may estimate the model using the most recent observations to generate the first set of forecast, e.g., , where represents the parameters estimated based on the observation window . The value of can be arbitrarily chosen given there are enough observations to estimate the model and there are enough variations in the explanatory variable. We then add more observations (e.g., one) to the estimation window and generate the second set of the forecast, e.g., and so forth. We may have sets of the forecasts, and we can calculate the final forecast as their average value:

(5)

In the supplementary material of this paper, we demonstrate how we can achieve more accurate forecasts with the IC method and the EWC method using simulation examples.

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2. The term ‘structural change’ is used interchangeably with the term of ‘structural break’ in the literature. In this study, we use the term “structural change” as in the retail context we expect the effect of the marketing activities to change gradually rather than in a sudden and abrupt way. We thank one of the anonymous reviewers to point this out. [↑](#footnote-ref-2)
3. We include in the supplementary material a simulation example with the intercept term to demonstrate the impact of the structural change on the forecasting performance. [↑](#footnote-ref-3)
4. We select the SKUs with positive movements for at least 90% of the time. [↑](#footnote-ref-4)
5. In Figure 6, the calendar events include Halloween, Thanksgiving, Christmas, New Year’s Day, President’s Day, Easter, Memorial Day, the 4th of July, and Labour Day. The promotional events include Feature and Display. [↑](#footnote-ref-5)
6. Huang et al. (2014) used alternative schemes such as Akaike’s Information Criterion. In this study, we find little difference in the results between different these two schemes. [↑](#footnote-ref-6)
7. We include the following US calendar events including *Halloween*, *Thanksgiving*, *Christmas*, *New Year’s Day*, *President’s Day*, *Easter*, *Memorial Day*, the *4th of July*, and *Labour Day*. [↑](#footnote-ref-7)
8. We reconduct the entire evaluation using a sequential Chow test for up to 70% of weeks and we little difference in the results. [↑](#footnote-ref-8)
9. We conduct the DM test based on all the error measures except the AvgRelMAE which does not fit into the framework of the DM test. [↑](#footnote-ref-9)
10. We refer these two periods as the promoted period and non-promoted period respectively. [↑](#footnote-ref-10)
11. The results for other forecasting horizons are similar and are not shown here for simplicity. [↑](#footnote-ref-11)
12. Other models including the Base-lift method, the ADL-own model, the ADL-own-EWC model, and the ADL-own-IC model are all outperformed by the four models in Table 5 and we do not show them for simplicity. [↑](#footnote-ref-12)
13. We choose to retain five factors based on the Scree plot and 77% of the original information have been retained. [↑](#footnote-ref-13)
14. In Table 6, we omit all small values for simplicity. [↑](#footnote-ref-14)
15. The results are consistent for other error measures and forecast horizons. [↑](#footnote-ref-15)
16. The results are similar for other forecast horizons. [↑](#footnote-ref-16)
17. For example, one of the alternative is to first make adjustments to the one-step-ahead forecast, and then calculate the two-step-ahead forecast based on the value of the one-step-ahead forecast which has adjusted, and so forth. [↑](#footnote-ref-17)